



**Optimal Fleet Replacement and Forecasting Under Uncertainty:  
The CP-140A Arcturus Maritime Surveillance Aircraft**

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## Outline

- Aim and scope
- Model assumptions
- Methodology
- Model details
- Results
- Summary and implications



## Aim and scope

- DND requested a study to help determine the most economic replacement time for a vehicle fleet
  - Decision Support
- *“In a world of intense global competition, simply setting appropriate general guidelines is not enough.”*  
(Hopp and Spearman, Factory Physics)



## Aim and scope

- **“Proof of principle”** new models for forecasting life cycle costs with the CP-140A – only three aircraft
- Nontrivial problem
  - (V. Greenfield and D. Presselin, RAND 2002)
  - (M. Dixon, RAND 2005)
  - (R.A. Pyles, RAND 2003)



## Aim and scope

- Fleet replacement – irreversible investment decision under uncertainty
- Future performance/costs unknown – stochastic evolution
- Incorporate effects of an unknown future – stochastic calculus



## Aim and scope

- **Attach a value to the capacity to wait**

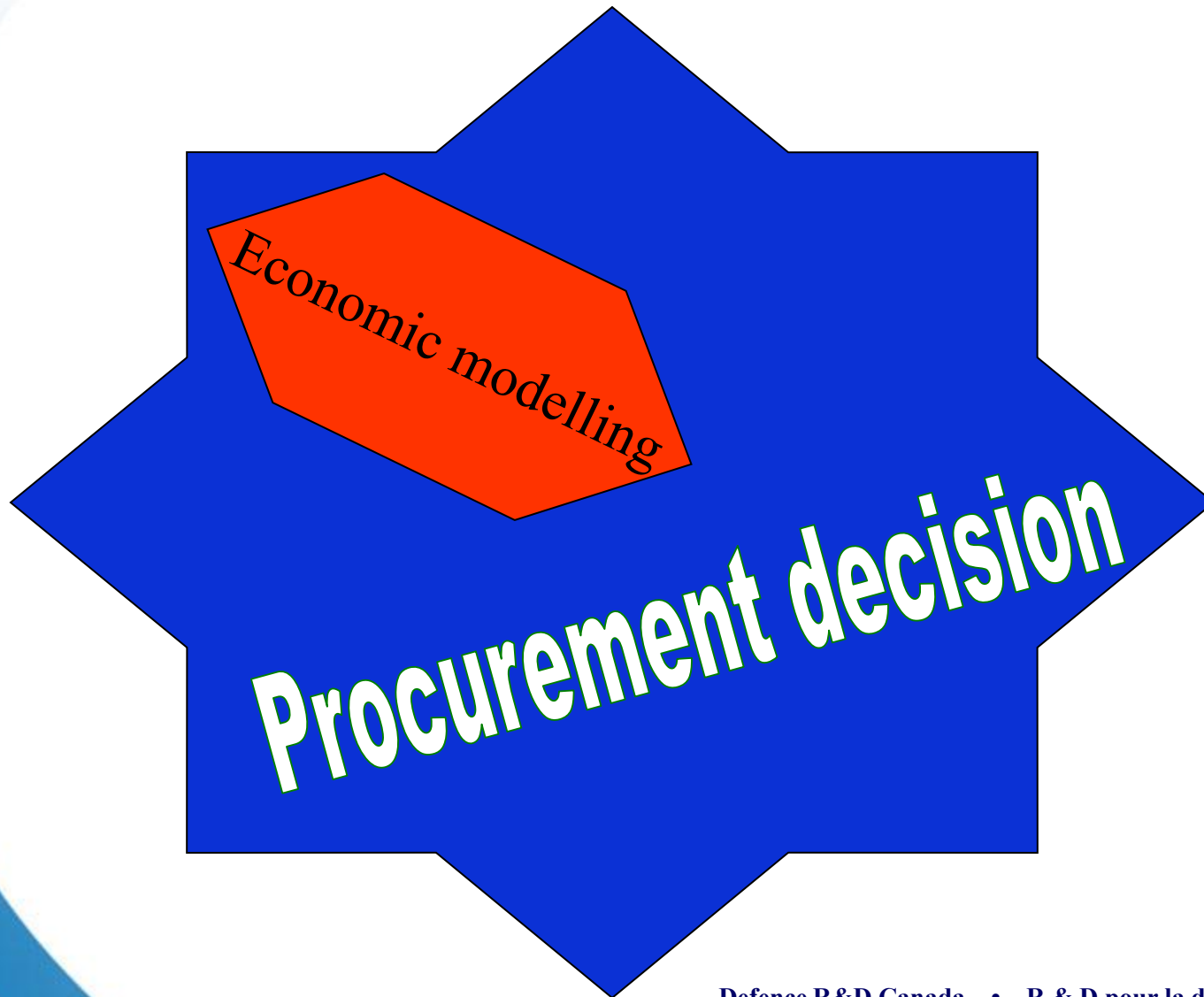


## Aim and scope

- Aircraft cost/performance optimization – active area of operations research
  - multilayered optimization software tools
  - Efforts to improve sustainability practices: **DRDC Air Vehicle R&D Program**, Advanced Repair and Life Enhancement – many fronts tackling problems in Life Cycle costing
- Insight can also be gained “Big Picture”



## Model assumptions





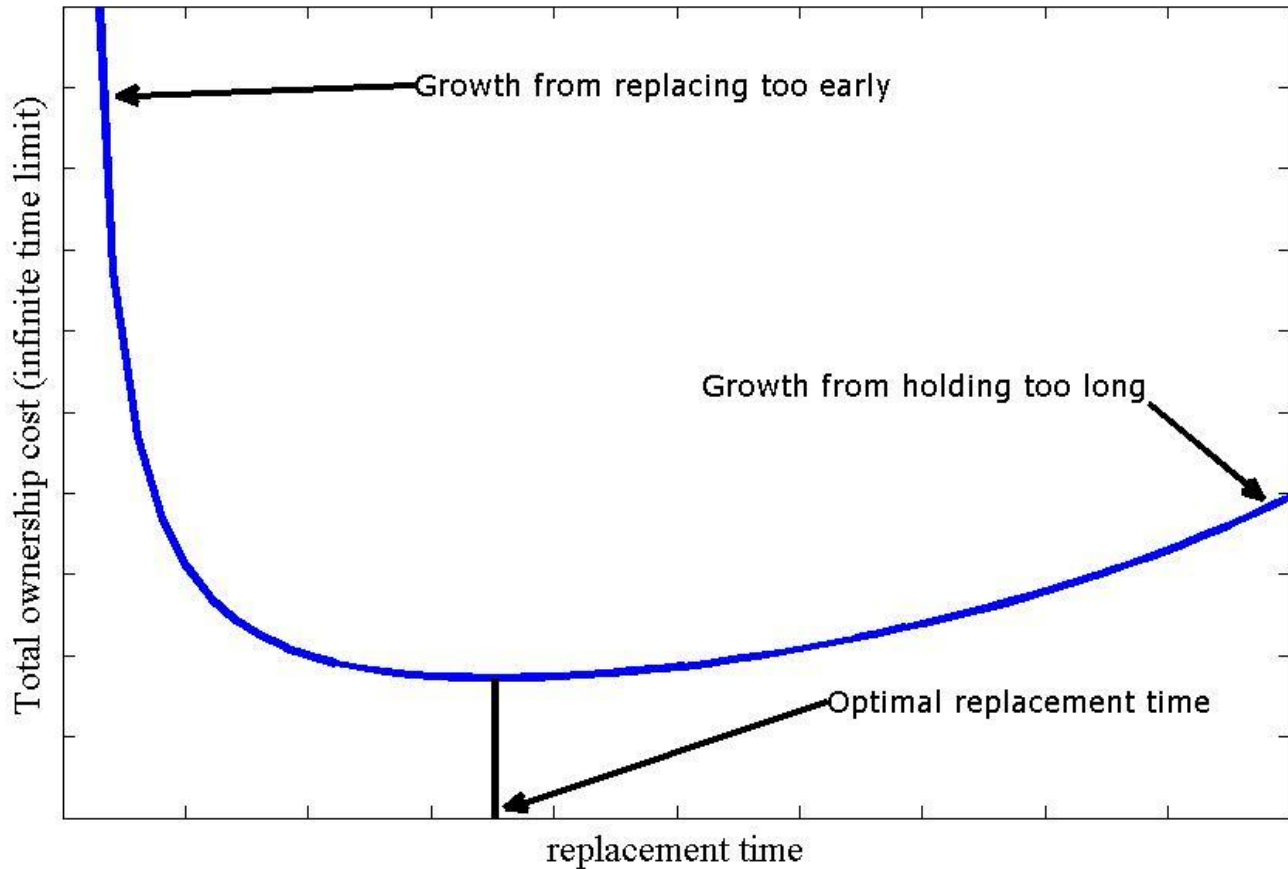


## Model assumptions

- Replacement choice **not** considered – **like-with-like comparison**
  - Infinite horizon generational replacement – for **specific vehicle**
  - Mathematical model creates like-with-like comparison
  - Discounting process used: 4.40% continuously compounding interest rate
- Costs do not include salaries or squadron support
- **Fleet becomes less resilient to random events with time**
- **Random events occur independently of vehicle life**
- **Age (less resiliency) leads to increasing O&M costs but also to less predictability**



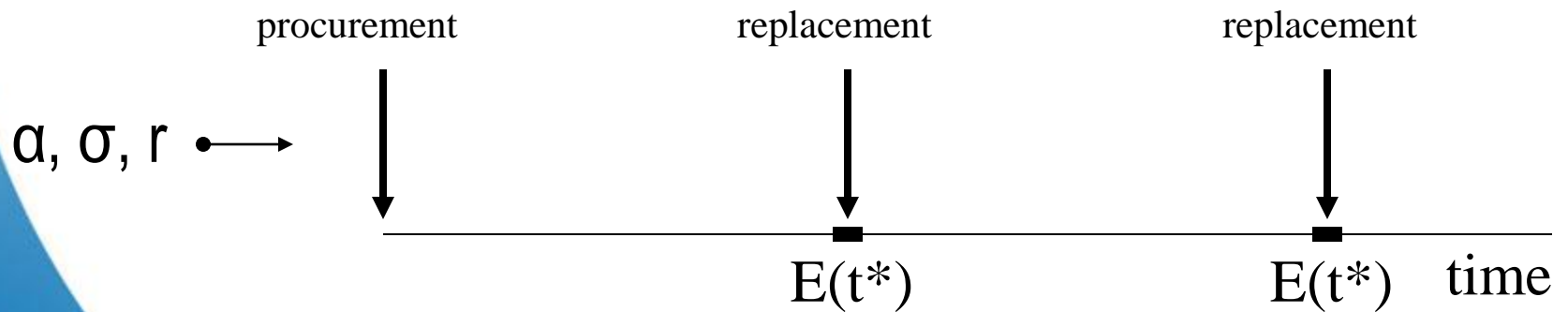
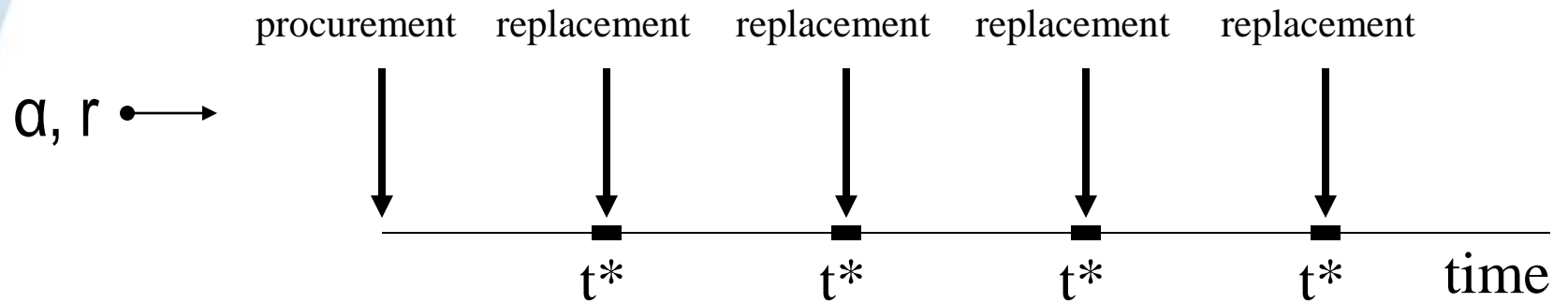
# Model assumptions





# Model assumptions

## Deterministic vs. Stochastic replacement model





## Model assumptions

- Stochastic (random) behaviour – model exponential growth in a stochastic background
- **Cost-benefit diffusion model (geometric random walk)** (Greenfield and Presselin, RAND 2002)
- No longer one-to-one relationship between optimal replacement time and annual O&M costs
- Must include a **measure of usefulness** – operational availability (Dixon and Keating, RAND 2003)



## Model details

### Stochastic case (Ito calculus):

O&M cost evolution:  
(geometric Brownian motion)

$$dm_u = \alpha m_u dt + \sigma m_u dW$$



Weiner element

General case (Stochastic differential equation):  $dx(t) = a(x, t)dt + b(x, t)dW$

**SDE → Partial differential equation for conditional probability density function**

Fokker-Planck equation:

$$\frac{\partial}{\partial t} p(x, t | x_0, t_0) = -\frac{\partial}{\partial x} (a[x(t), t] p(x, t | x_0, t_0)) + \frac{1}{2} \frac{\partial^2}{\partial x^2} (b^2[x(t), t] p(x, t | x_0, t_0))$$



## Model details

$$a(m_u^*, m_u) = dt + (a(m_u^*, m_u) + \mathbf{E}(da(m_u^*, m_u)))$$

- Recursive Bellman relation for replacement time

(Greenfield and Presselin, RAND 2002)

- Resulting Fokker-Planck equation
  - expectation time in a given region

$$\frac{1}{2} \sigma^2 m_u^2 \frac{d^2 a(m_u^*, m_u)}{dm_u^2} + \alpha m \frac{da(m_u^*, m_u)}{dm_u} + 1 = 0$$



## Model details

- Recursive Bellman relations for expected replacement time, expected discount factor, and expected net present value (Greenfield and Presselin, RAND 2002)
- Combined relationships yield an expected ownership cost curve which can be minimized to determine the cutoff barrier  $m_u^*$  (can impose the cutoff from exogenous policy considerations as well)



## Model details

Deterministic replacement time:

$$a(m_u^*, b) = \frac{1}{\alpha} \ln(m_u^*/b)$$

Stochastic replacement time:

$$a(m_u^*, b) = \frac{\ln(m_u^*/b)}{\alpha - (1/2)\sigma^2}$$





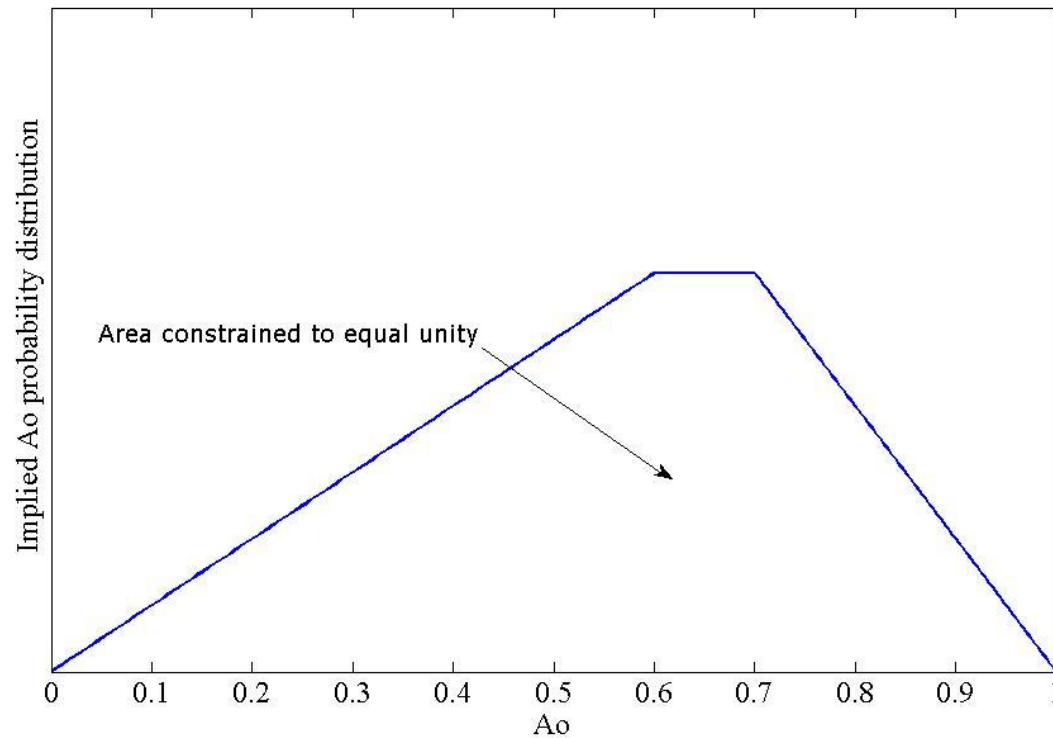
## Model details

- Model extension: replacement time depends on utility
  - Military requires a performance measure
  - Model requires consensus on the utility measure
  - Operational Availability ( $A_o$ ) used as proxy for “usefulness”
  - Examine ratio process, approximate by geometric random walk



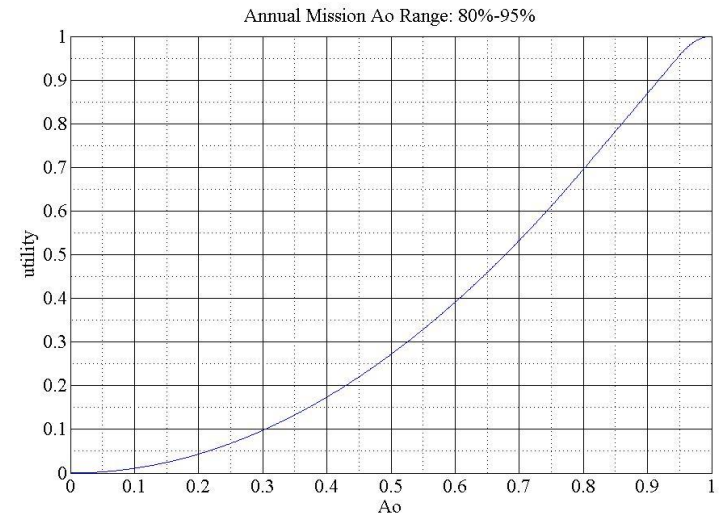
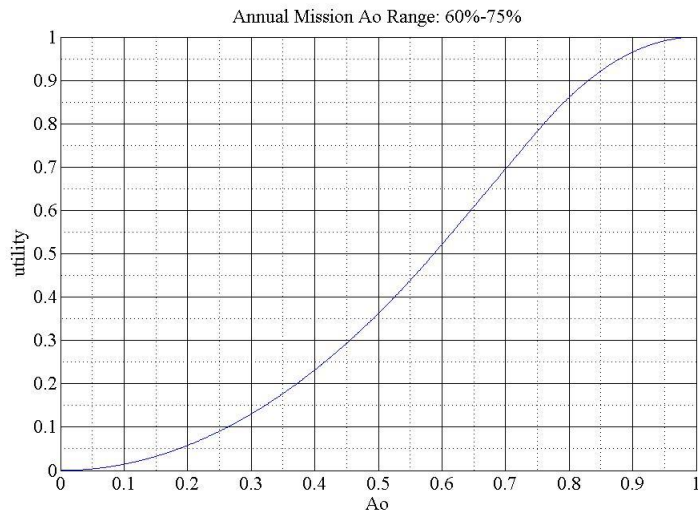
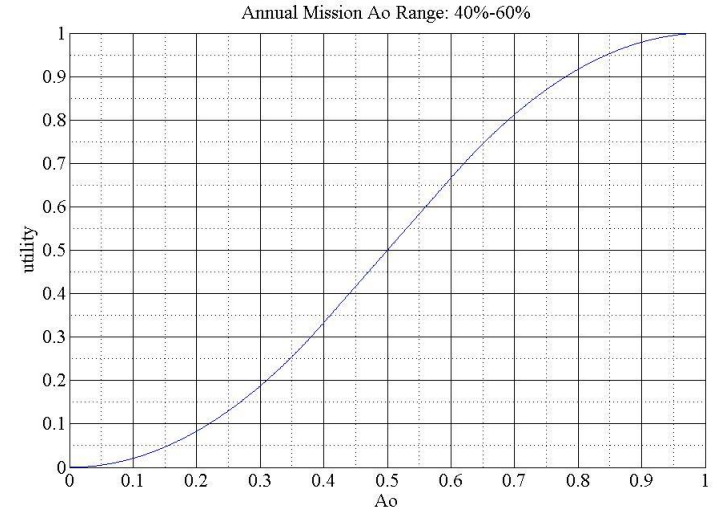
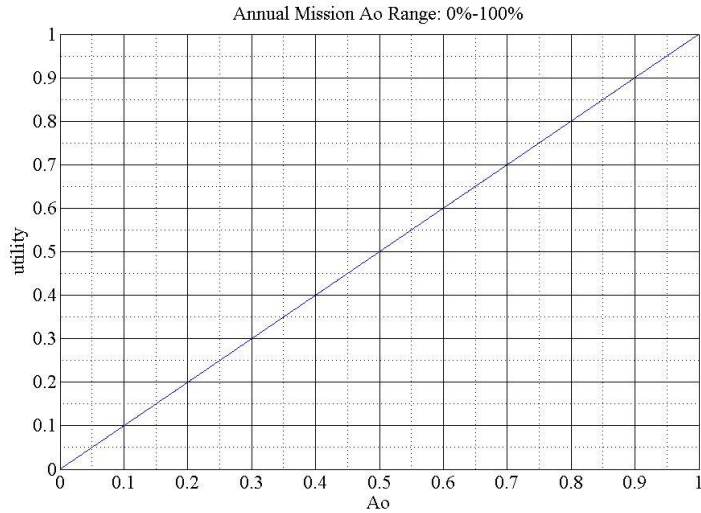
# Model details

Effectiveness measure: implied utility





# Model details



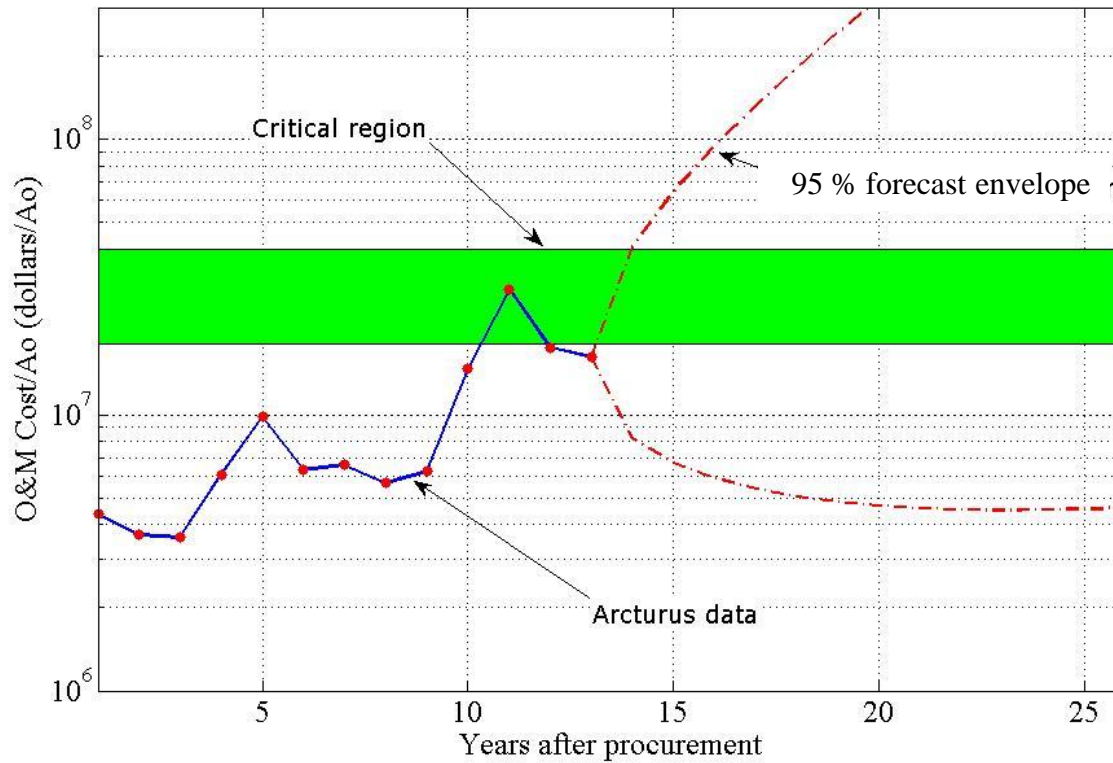


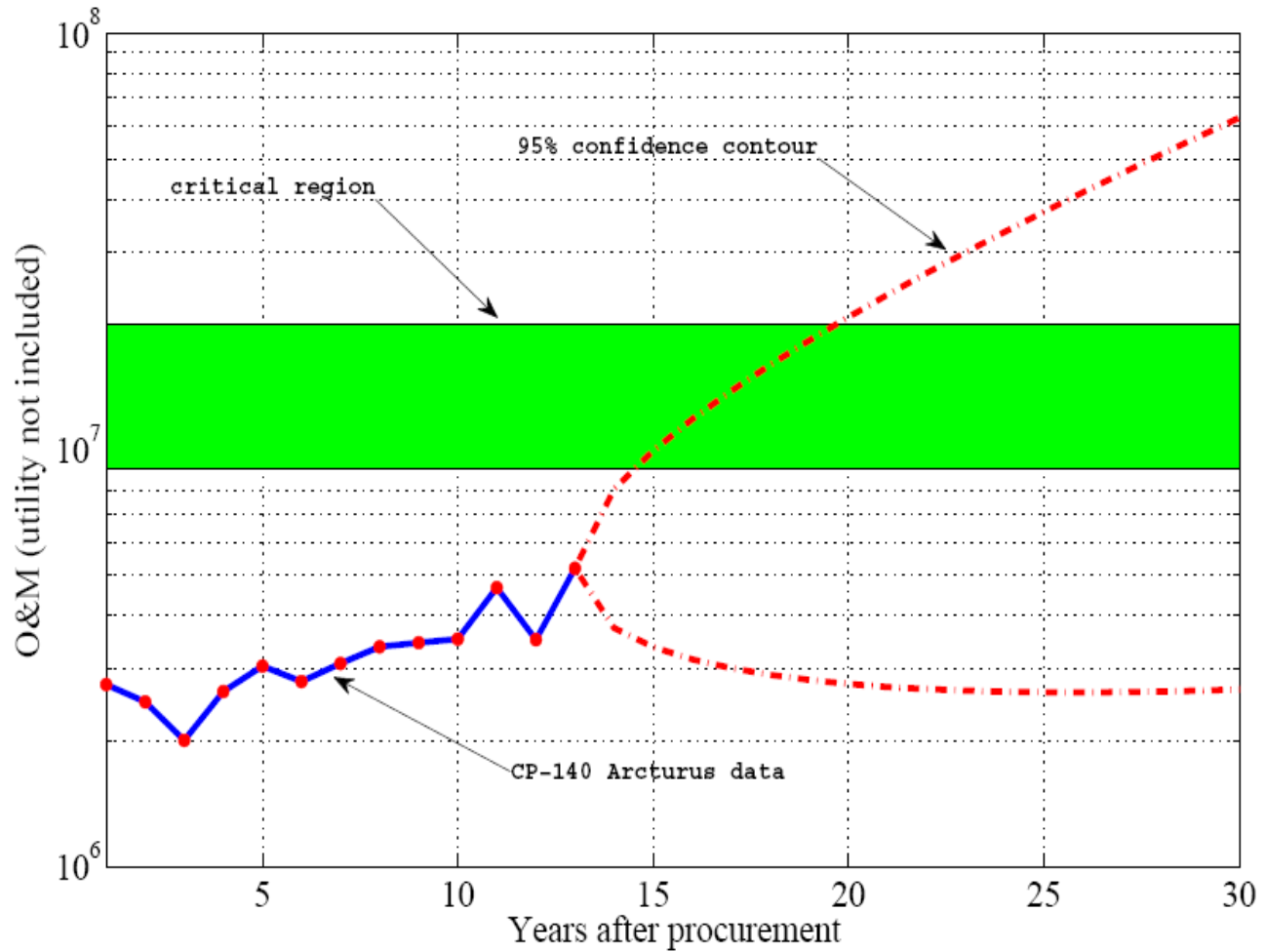
## Results

- Theoretically (parameters known with infinite precision),  $m_u^*$  is a barrier
- Data has uncertainty – must use error analysis – let the data be the guide
  - Prototype study on system with low amount of data
  - Accommodate uncertainty by “smearing” the barrier with MLE of parameters with error bars
  - Future research: systems with more data, exploit numerical bootstrapping methods (iterated conditioning)
- Data uncertainty:  $m_u^*$  becomes a region:
  - *Critical Region: **Give the decision maker a decision tool***



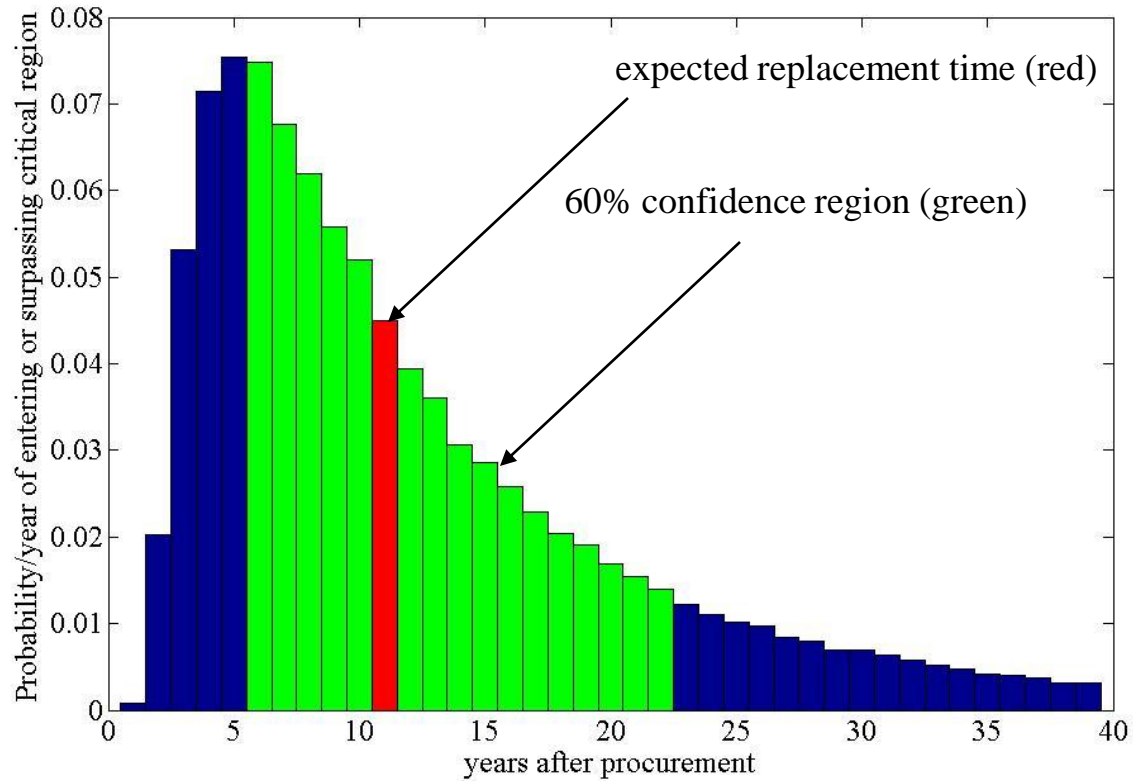
# Results







# Results





## Results

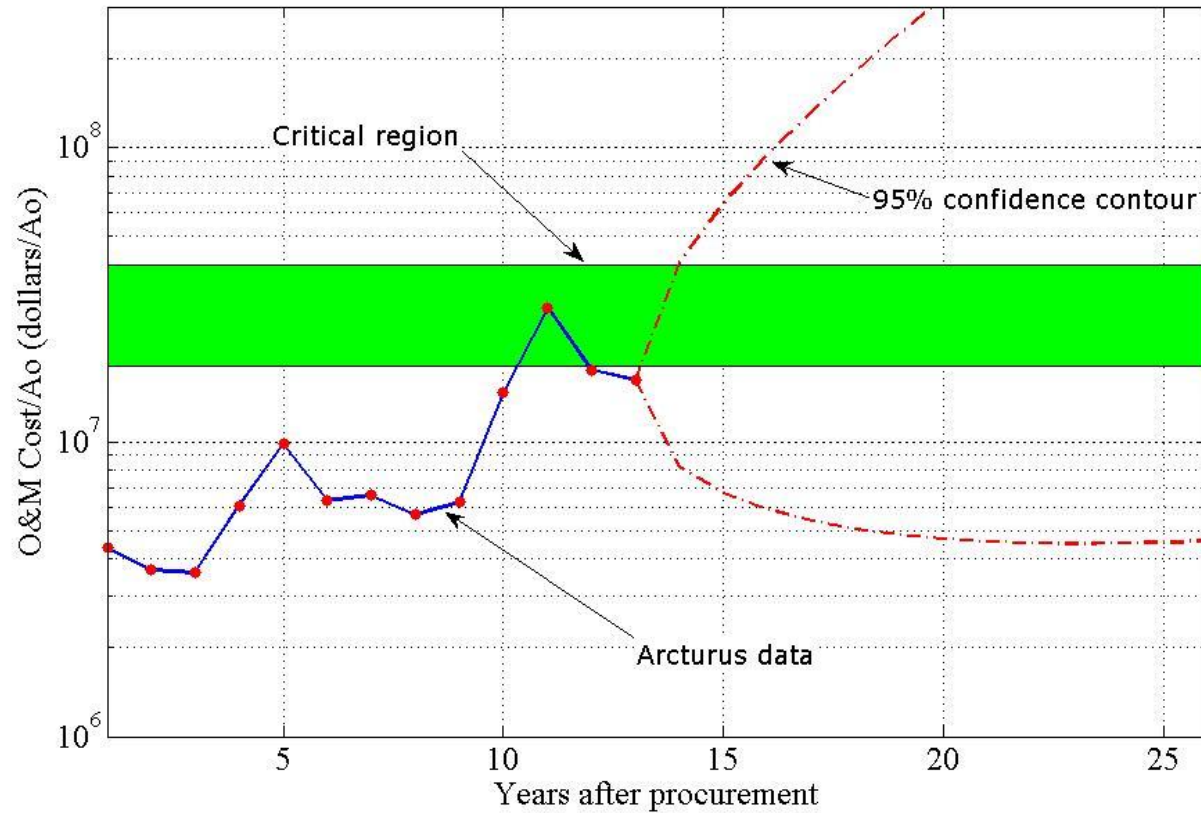
- Expected sojourn time within critical region – help decision makers understand delay costs/benefits
- Generalize the boundary conditions for FPE associated with the expectation time

$$T(\tilde{m}_u) = \frac{1}{k_2} \left[ \frac{(\tilde{U} - \tilde{L})(e^{\frac{-k_2}{k_1}\tilde{U}} - e^{\frac{-k_2}{k_1}\tilde{m}_u})}{\left( e^{\frac{-k_2}{k_1}\tilde{L}} - e^{\frac{-k_2}{k_1}\tilde{U}} \right)} - \tilde{m}_u + \tilde{U} \right]$$





# Results





## Results

- Expected time has large variance, but can calculate probability of entering critical region from any given time step
- In this example (CP-140A):
  - deterministic “age”: 8 years (with utility),
  - stochastic “age” (with utility): 11 years (60% bounds, 6 years to 22 years)
  - Stochastic “age” (without utility): 23 years
- Use critical region as a decision tool



## Summary and conclusions

- Stochastic processes can give longer expectation times for replacement, includes random events
- Once fleet enters the **critical region – procurement process, reset-the clock overhaul, or fleet changes** should be considered
- Can calculate the probability (with utility) of entrance, exit or sojourn times in the critical region from any given time step – help set replacement time
- Model implicitly attaches **value to the capacity to delay** a decision – sunk costs cannot be recovered, waiting has value
- Tool applicable to wide variety of vehicle fleets – **less resilient → higher O&M costs from random events, higher level of unpredictability**



## Summary and conclusions

- Model refinements
  - Model the  $A_0$  process (stochastic mean reversion, reflected mean reversion, etc.)
  - Correlations in  $A_0$  and costing data
  - Departures from geometric random walk (coupled stochastic differential equations)
  - Bootstrap methods (iterated conditioning)



## Summary and implications

Possible emergent environment picture:

